Dynamic Global Packet Routing in Wireless Networks

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Abstract

We consider schemes for reuse-efficient packet access in wireless data networks. We show that computing the maximum ergodic packet arrival rate is NP-hard. We give an upper bound on the maximum ergodic throughput in terms of the eigenvalues of matrices related to the path-gain matrix. We present simple, practical heuristic algorithms which exhibit good throughput and packet delay and report on results of preliminary simulations. More sophisticated algorithms that yield optimal throughput are also presented. A recent result of McKeown, Anantharam and Walrand on scheduling of input-queued switches is obtained as a byproduct.

1. Introduction

This paper addresses schemes for reuse-efficient transmission in wireless packet data networks. Particular attention is paid to recent proposals (see for example [19]) for wireless data packet networks in which the medium access (MAC) layer protocol is:

(i) deterministically scheduled.

(ii) has good delay performance.

(iii) provides near-optimal channel capacity in a single channel (i.e. is reuse-efficient).

References [11, 10] provide an overview of some of the salient themes and constraints of particular relevance in the design of wireless packet data networks. Because of the limited availability of the spectrum, *interference management and power control* are crucial to obtain optimum system capacity in wireless networks. Routing in a wireless network involves both *spatial* and *temporal* allocation of resources to meet given service demands while respecting *quality of service* (QoS) constraints such as maximum packet delay. Such QoS guarantees are part of ATM (Asynchronous Transfer Mode) contracts. An additional concern in wireless data networks is *fading*, i.e. the degradation of received signal power at the receiver. Due to the nature of the channel, the deliverable payload of a wireless data channel is not all that high. Current estimates lie in the range of 1 to 10 Mbps, in the optimistic case.

Throughput can also be severely limited by the fact that multiple users are trying to use a given channel at the same time. This is well known from experience with ALOHA and the Carrier-Sense-Multiple-Access CSMA protocols typically used in wired and wireless local area networks (LAN's). ALOHA suffers from poor throughput while CSMA can be sensitive to thresholds for collision detection as well as the choice of methods for collision-resolution [18, 1].

We study in this paper an alternate approach to ALOHA and CSMA in which multiple stations with packets ready to transmit can determine ahead of time whether the signal-to-interference ratio (SIR) required for adequate bit-error-rate (BER) is obtainable in the next transmission interval. Thus channel access is conflict-free with good BER performance. However the scheduling of stations must be done carefully to get good throughput as well. This approach can be applied to either a time-division multiple access (TDMA) system or a channel-division multiple access (CDMA) system.

Related scheduling problems have been studied for input-queued switches (see [14] and references therein). Scheduling problems where requests that cannot be served are *blocked and cleared* rather than being queued have been analyzed in [7, 15, 6].

2. Contribution

This paper presents new bounds on the available throughput in a fixed wireless data network. It is shown that the problem of determining the maximum stable throughput for a system with unbounded buffers is NP-hard. However, we address this problem by considering a variety of practical algorithms which obtain good throughput, good signal-to-interference protection and are easy to implement, even for systems with hundreds of links for which simultaneous access is desired (a *link* is an ordered pair of stations, the first of which is the *transmitter*, the second being the *receiver*). Our algorithms might be adapted to cases where there are mobile terminals moving within a service area but, for simplicity, we only consider a wireless network in which all the terminals are fixed. We study the case of a single channel; extension of ideas presented here to multiple channels is possible.

We consider two modes of operation. In the first, discrete interference relationships between every pair of links are determined before transmission and are assumed to remain constant. That is, the interference constraints are represented by an undirected graph in which the nodes are *links* and any edge between two links exist if both links may not be in simultaneous operation. In the second mode, *aggregate interference* is taken into account and the controller has detailed information on the propagation path losses between any station pair; *power control* may or may not be used. With power control, transmitter powers may be varied in order to alter the interference environment and influence the SIR's. It is known that in the case where power control is possible significant capacity gains may be realized, though at the cost of a more difficult interference management problem. Our theoretical results on the maximum stable throughput apply in either context.

We assume that all scheduling is done by a central controller; distributed implementations are currently under study. The central controller is aware of all routing requests and packet queue states at every instant, and all conflict relationships between links or the actual path-gains. This model is quite reasonable for a moderate number of stations (100 - 200), as is the case for current systems. Indeed currently deployed AMPS (Advanced Mobile Phone Service) wireless systems integrate the management of up to 200 stations (cell sites) in a single service center known as a mobile switching center (MSC).

Our algorithms can also be used for other scheduling problems in which multiple requests contend for the same resources. In particular, using a Lemma in [14], we note in Section 6 that a recent result of McKeown, Anantharam and Walrand [14] on scheduling of input-queued switches follows from our Theorem 6.1. Our proof is similar but simpler than [14].

3. The Graph Model

Let $\mathbf{G} = [G_{ij}]$ denote the path-gain matrix, where G_{ij} equals the path-gain from the transmitter of link i to the receiver of link j. At a given time-step, let p_i be the power level of transmitter of link i (in general p_i is a function of the *link* i, not only of the transmitter of link i): $p_i > 0$ if and only if link i is active. Thus, the interference signal received by the receiver of link i is $\sum_{j \neq i} G_{ji} p_j$, and the power received by the receiver of link i from the transmitter of link i is $G_{ii} p_i$. Let m be the number of links, γ_i be the required signal-to-interference ratio (SIR) for link i, and $\Gamma = diag(\gamma_1, \ldots, \gamma_m)$ be the diagonal matrix with the γ_i's on its diagonal.

Let N be any real matrix with nonnegative off-diagonal entries. It follows from the theory of non-negative matrices [17] that one eigenvalue of N that has the largest real part is real (and uniquely determined). It will be denoted by $\lambda_0(N)$.

Let $\mathbf{G}' = [G'_{ij}]$ where $G'_{ij} = G_{ij}/G_{ii}$ for $i \neq j$, and $G'_{ii} = 0$. A set of links S can be activated (i.e. is *feasible*) at a given time-step if and only if there exists a power vector $\mathbf{p}$ whose support is S (i.e. $p_i > 0$ if and only if $i \in S$) such that $\Gamma \mathbf{G}' \mathbf{p} + \nu \leq \mathbf{p}$, where ν is a noise vector, and element-wise comparison is implied. In other words, the SIR at any active link is above the SIR threshold. Assuming that the power vector can be arbitrary (i.e. there is no upper constraint on the transmitter power), this is equivalent (see [16, 20] and references therein) to the condition $\lambda_0(\mathbf{G}'_S - \Gamma_S^{-1}) < 0$, where $\mathbf{G}'_S$ is the submatrix induced by the elements of S. Let $H = \{S_1, \ldots, S_h\}$ be the collection of feasible subsets of the links. Note that any subset of a feasible set, including the empty set, is feasible.

Each link i has a queue associated with it. We assume that packets arrive at link i according to a Poisson distribution of rate r_i. When a packet arrives at link i, it is appended to the tail of the queue at that link. When a set of links S is scheduled at a time-step, the head of the queue of each link in S gets removed from the queue.

4. Stability

Let $a_i(t)$ be the number of packets generated at link i at time t, and $a(t) = (a_i(t))$ be the arrival vector. Let $r = (r_i)$ be the vector of arrival rates.

Theorem 4.1. *If there is a scheduling algorithm for which the system is ergodic, then r is strictly dominated by a convex combination of the characteristic vectors of the S_k's. Conversely, if r is strictly dominated by a convex combination of the characteristic vectors of the S_k's, then there exists a scheduling algorithm for which the system is ergodic.*

Proof Assume first that there exists a scheduling algorithm for which the system is ergodic. Let $q_i(t)$ be the queue-size of link i at time t, and $q(t) = (q_i(t))$ be the vector of queue-sizes. Let $S(t)$ be the feasible subset scheduled at time t, and $\chi_{S(t)}$ be its $0-1$ characteristic vector. We have

$$q(t) = a(0) + a(1) + \cdots + a(t-1) - tv, \qquad (4.1)$$

where $v = \frac{1}{t}(\chi_{S(0)} + \chi_{S(1)} + \cdots + \chi_{S(t-1)})$. Since v belongs to the set C of convex combinations of the characteristic vectors of the S_k's, so does its expected value. Thus, it follows from Eq. 4.1 that $r - \mathrm{E}[q(t)]/t \in C$. By taking the limit as t goes to infinity, we conclude that $r \in C$, since C is a closed set.

We now show that r is strictly dominated by a vector in C. The total arrival at link i during the interval $[0, t-1]$ has a Poisson distribution with mean $r_i t$. Let B_i be the event that it exceeds $r_i t + \sqrt{r_i t}$. The probability of B_i is at least an absolute constant c. If follows from Eq. 4.1 that

$$\mathrm{E}[q(t)|B_1, \ldots, B_m] \geq tr + \sqrt{t}\sqrt{r} - t\mathrm{E}[v|B_1, \ldots, B_m], \qquad (4.2)$$

where $\sqrt{r}$ is the vector $(\sqrt{r_i})$. The left-hand side of Eq. 4.2 is upper bounded by $\mathrm{E}[q(t)]/c^m$, which is bounded by a constant independent of t. By choosing t sufficiently large, we conclude that r is strictly dominated by the vector $\mathrm{E}[v|B_1, \ldots, B_m]$, which belongs to C.

Conversely, we show that if $\mathrm{E}[a]$ is strictly dominated by a vector in C, then there is a scheduling algorithm for which the system is ergodic. Assume that $(1+\epsilon)r \leq \sum_{k=1}^{h} \alpha_k \chi_{S_k}$, with $\alpha_k \geq 0$ and $\sum_{k=1}^{h} \alpha_k = 1$. At each time-step, schedule subset S_k with probability α_k. Then the system is ergodic and the expected queue-size of each link is $O(1/\epsilon)$. This is because the arrival rate at link i is r_i and the departure rate, when link i is nonempty, is at least $(1+\epsilon)r_i$. Our claim then follows from standard queuing theory [8]. ■

Corollary 4.2. *For general G, (γ_i) and r, determining whether there is a scheduling algorithm for which the system is ergodic is NP-hard.*

Proof If G is the adjacency matrix of a graph K, and if $\gamma_i = 1/2$ for all i, then the feasible subsets are the independent sets of K. If $r_i = s$ for all links i, then Theorem 4.1 implies that there is a scheduling algorithm for which r is ergodic if and only if $1/s$ is greater than the fractional chromatic number of K. The fractional chromatic number of a graph is the minimum sum of nonnegative coefficients $\beta_1, \cdots, \beta_h$ such that, if we assign β_k to independent set S_k, the sum of values assigned to the independent sets containing w is at least 1 for each vertex w. The fractional chromatic number of a graph is known to be NP-hard to compute [13]. ■

5. Upper bound on the maximum ergodic throughput

We give in this section a necessary, but not sufficient, condition to determine whether there is a scheduling algorithm for which the system is ergodic. This condition can be checked in polynomial time.

The eigenvalues of any real symmetric $m \times m$ matrix M are known to be real, and will be denoted by $\lambda_0(M) \geq \lambda_1(M) \geq \ldots \geq \lambda_{n-1}(M)$.

Theorem 5.1. *If there is a scheduling algorithm for which the system is ergodic, then*

$$\lambda_0\left(R - \frac{1}{\lambda_{n-1}(R^{-1}B)}B\right) \leq 1,$$

where R is the diagonal matrix representing the vector r, and $B = (\min(G'_{ij}, G'_{ji})) - \Gamma^{-1}$.

Proof Since the matrices B and $G' - \Gamma^{-1}$ have nonnegative off-diagonal entries and B is dominated by $G' - \Gamma^{-1}$, $\lambda_0(B_S) \leq \lambda_0(G'_S - \Gamma_S^{-1})$ for any subset S of links [17]. In particular, $\lambda_0(B_S) < 0$ if S is feasible. As noted before, $r = \sum_k \alpha_k \chi_{S_k}$, where the α_k's are nonnegative and sum up to 1. Let $x = -1/\lambda_{n-1}(R^{-1}B)$, where $\lambda_{n-1}(R^{-1}B)$ is the smallest eigenvalue of $R^{-1}B$. Since the diagonal elements of B are negative, $x > 0$. Let f be an eigenvector corresponding to the largest eigenvalue of $R + xB$, and f_k its projection on S_k. That is, the ith coordinate of f_k is equal to the ith coordinate of f if $i \in S_k$, and is equal to 0 otherwise. The smallest eigenvalue of $I + xR^{-1}B$ is 0, by definition of x. The matrices $I + xR^{-1/2}BR^{-1/2}$ and $I + xR^{-1}B$ have the same eigenvalues since $I + xR^{-1/2}BR^{-1/2} = R^{1/2}(I + xR^{-1}B)R^{-1/2}$, and so $I + xR^{-1/2}BR^{-1/2}$ is positive semi-definite. Hence the matrix $R + xB$ is also positive semi-definite, since $R + xB = R^{1/2}(I + xR^{-1/2}BR^{-1/2})R^{1/2}$. It follows that the quadratic form $< g, f >= g \cdot (R + xB)f$ is positive semi-definite. The Cauchy-Schwartz inequality with respect to this positive semi-definite form implies that $< g, h >^2 \leq < g, g >< h, h >$, for any m-dimensional real vectors g and h. Taking $g = R^{-1}f_k$ and $h = f$ we get

$$\begin{aligned}
&(R^{-1}f_k \cdot (R + xB)f)^2 \\
&\quad \leq (R^{-1}f_k \cdot (R + xB)R^{-1}f_k)(f \cdot (R + xB)f) \\
&\quad = (f_k \cdot R^{-1}f_k + xR^{-1}f_k \cdot BR^{-1}f_k)\lambda_0(R + xB)||f||^2 \\
&\quad \leq (f_k \cdot R^{-1}f_k)\lambda_0(R + xB)||f||^2.
\end{aligned}$$

The third inequality follows from the inequality $\lambda_0(B_{S_k}) < 0$, and thus $R^{-1}f_k \cdot BR^{-1}f_k \leq 0$. But

$$\begin{aligned}
R^{-1}f_k \cdot (R + xB)f &= \lambda_0(R + xB)R^{-1}f_k \cdot f \\
&= \lambda_0(R + xB)R^{-1}f_k \cdot f_k.
\end{aligned}$$

Together with the preceding equation, this implies that

$$(f_k \cdot R^{-1}f_k)\lambda_0(R + xB) \leq ||f||^2.$$

Since

$$\begin{aligned}
\sum_k \alpha_k(f_k \cdot R^{-1}f_k) &= \sum_k \alpha_k(f_k \cdot R^{-1}f) \\
&= Rf \cdot R^{-1}f \\
&= ||f||^2,
\end{aligned}$$

we conclude that $\lambda_0(R + xB) \leq 1$, as desired. ■

Corollary 5.2. *If all arrival rates are equal to r and if the system is ergodic, then*

$$r \leq \frac{\lambda_{n-1}(B)}{\lambda_{n-1}(B) - \lambda_0(B)}. \tag{5.1}$$

Remark. The above inequalities hold when B is any non-null symmetric matrix such that $0 \leq (B + \Gamma^{-1})_{ij} \leq \min(G'_{ij}, G'_{ji})$, for all i, j. This is because this condition implies that $\lambda_0(B_S) \leq \lambda_0(G'_S - \Gamma_S^{-1}) < 0$. In our experiments, we have chosen $B_{ij} = (\min(G'_{ij}, G'_{ji}, (\gamma_i\gamma_j)^{-1/2})) - \Gamma_{ij}^{-1}$.

When r is constant across links and we consider only pairwise interference, i.e. when the feasible subsets are the independent sets of a graph K, Eq. 5.1 is a slightly stronger version of Hoffman's bound (see [2], and [9] for related work) on the chromatic number. Indeed, we can model this case by setting G' to be the adjacency matrix of K and setting $\Gamma = \epsilon^{-1}I$, where $0 < \epsilon < 1$. By letting ϵ tend to 0, we get the bound

$$r \leq \frac{\lambda_{n-1}(K)}{\lambda_{n-1}(K) - \lambda_0(K)}. \tag{5.2}$$

Using results from [12, 4], it can be shown that the bound in Eq. 5.1 can be off by a factor of at least $\Omega(m/2^{O(\sqrt{\log m})})$ from optimal, for general G and Γ, even if B is chosen optimally. However, in the simulations we performed, the bound given by Eq. 5.1 was off by only a small multiplicative factor from optimal, as discussed in Section 7.

6. Algorithms and Heuristics

In a real-life application, time slot lengths are likely to be in the range of 10's of msecs. This places a bound on the complexity of algorithms for practical implementation. A natural algorithm is to schedule a largest feasible subset at each time-step. However, there are simple examples (i.e. when the interference graph is a large star and the arrival rates are slightly smaller than .5) where this algorithm yields sub-optimal throughput and some stations may be starved for long periods. Consequently packet delays will be high.

We present below two algorithms that yield optimal throughput with good delay performance. However, they are difficult and impractical to implement. The remaining algorithms achieve good (albeit, in general, sub-optimal) throughput with good delay performance. Furthermore, they are relatively easy to implement.

Algorithm Max-Weight: at each time-step, schedule a maximum weighted feasible subset. The weight of a link can be its queue-size (s-Max-Weight), the waiting time of the oldest packet in its queue (w-Max-Weight), or any other potential function. By convention, the waiting time at time t of a packet generated at time s is $t - s + 1$.

We show that both Algorithm s-Max-Weight and Algorithm w-Max-Weight give the optimal stable throughput. Furthermore, the ergodic delay under Algorithm w-Max-Weight if off from the optimal ergodic delay by a factor of order m plus an additive factor (see Theorem 6.2 for a detailed statement.) For typical networks, however, we suspect that the behavior of Algorithm w-Max-Weight is much closer to "optimality" than what is implied by Theorem 6.2. The proof of these results for Algorithm w-Max-Weight is omitted in this abstract for lack of space, but will appear in the journal version of the paper. We note that, unfortunately, it is NP-hard to find a maximum weighted feasible subset in general.

Let $S_{k(t)}$ be the subset scheduled by Algorithm Max-Weight at time t, and $q'(t) = q(t) + a(t)$.

Theorem 6.1. *If r is such that there is an algorithm for which the system is ergodic, then algorithm s-Max-Weight achieves ergodicity.*

Proof By Theorem 4.1, $r < r' = \sum_{k=1}^{h} \alpha_k u_k$, where $u_k = \chi_{S_k}$, $\alpha_k \geq 0$ and $\sum_{k=1}^{h} \alpha_k = 1$. For any time-step t, $q(t+1) = q'(t) - u_{k(t)}$.

$$\begin{aligned} ||q(t+1)||^2 &= ||q'(t) - u_{k(t)}||^2 \\ &= ||q'(t)||^2 + ||u_{k(t)}||^2 - 2q'(t) \cdot u_{k(t)}. \end{aligned}$$

On the other hand, by definition of the algorithm, $k(t)$ is chosen so that $q'(t) \cdot u_{k(t)}$ is maximum. As a consequence,

$$\begin{aligned} q'(t) \cdot u_{k(t)} &\geq \sum_{k=1}^{h} \alpha_k q'(t) \cdot u_k \\ &= q'(t) \cdot r'. \end{aligned}$$

Combining the two preceding equations yields

$$\begin{aligned} &||q(t+1)||^2 \\ &\leq ||q'(t)||^2 + ||u_{k(t)}||^2 - 2q'(t) \cdot r' \\ &= ||q(t)||^2 + ||a(t)||^2 + 2q(t) \cdot a(t) + ||u_{k(t)}||^2 \\ &\quad -2q'(t) \cdot r' \\ &\leq ||q(t)||^2 + ||a(t)||^2 + 2q(t) \cdot (a(t) - r') + m. \end{aligned}$$

Taking expectations and noting that $q(t)$ and $a(t)$ are independent, it follows that

$$\begin{aligned} \mathrm{E}\left[||q(t+1)||^2\right] &\leq \mathrm{E}\left[||q(t)||^2\right] - 2\mathrm{E}\left[q(t)\right] \cdot (r' - r) \\ &\quad + \mathrm{E}\left[||a(t)||^2\right] + m, \end{aligned}$$

and so

$$\begin{aligned} &2(r' - r) \cdot \mathrm{E}\left[q(t) + q(t-1) + \cdot + q(0)\right] \\ &\quad \leq (t+1)\left(\mathrm{E}\left[||a||^2\right] + m\right). \end{aligned}$$

Since $r < r'$, this implies that the expected value of $(q(t) + q(t-1) + \cdots + q(0))/(t+1)$ is bounded, and thus the system is ergodic. ∎

Theorem 6.2. *If r is such that there is an algorithm for which the system is ergodic, then Algorithm w-Max-Weight achieves ergodicity. The ergodic waiting time under algorithm w-Max-Weight is off the ergodic waiting time under any algorithm for which the system is ergodic by at most a multiplicative factor of order m plus an additive factor of order $m/r \cdot \mathbf{1}$.*

Proof Omitted for lack of space. ∎

The results in this section also apply for other scheduling problems where the feasible subsets are drawn from a given collection H. In particular, for input-queued switches, H consists of all matchings. It is shown in [14] that for *admissible arrival rates*, the algorithm that schedules a maximum weighted matching (where the weight is the queue-size) achieves 100% throughput (see [14] for details). The maximum weighted matching coincides with Algorithm s-Max-Weight in this setting. Thus, the result in [14] follows from Theorem 6.1. This is because, as shown in Lemma 1 in [14], an admissible rate vector is a convex combination of characteristic vectors of matchings. It follows from Theorem 6.2 that the maximum weighted matching algorithm where the weights are the waiting times of the oldest packet in queue also achieves 100% throughput.

Note that if the vectors r and (γ_i) and the matrix G are known in advance, it is possible to find a vector $r' \in C$ that strictly dominates r using linear programming. For general G and (γ_i), this might take an exponential amount of preprocessing time, though. Since H lies in a vector space of dimension m, r' can be written as a convex combination of at most $m+1$ characteristic vectors of feasible subsets. These $m+1$ subsets can be determined using an exponential amount of preprocessing time. Once these subsets are determined, it is easy to run a variant version of Algorithm Max-Weight by choosing the subset of maximum weight among these $m+1$ subsets. Theorems 6.1 and 6.2 also hold for this version of Algorithm Max-Weight.

An exponential bound with a small decay rate can be shown to hold on the tail of the distribution of the queue-sizes of Algorithms s-Max-Weight and w-Max-Weight. We show below that, for a suitable weight function, a variant of Algorithm Max-Weight gives an exponential bound on the tail of the distribution of the queue-sizes with a better decay rate. This algorithm assumes that, for a given $\epsilon > 0$, there is a scheduling algorithm for which the system is stable under the arrival vector $(1+\epsilon)r$. An algorithm of similar flavor has been considered [6] for routing in ATM networks.

For fixed $\alpha = \Theta(\epsilon)$ to be determined later, let $\phi(x) = e^{\alpha x}$ and $\Phi(q) = \sum_i \phi(q_i)$.

Algorithm Exp-Weight: At time step t, schedule a feasible subset so that $\Phi(q(t+1))$ is minimized.

As in the case for Algorithm Max-Weight, Algorithm Exp-Weight is NP-hard to implement.

Theorem 6.3. *Assume that r is such that there is an algorithm for which the system is ergodic under the arrival vector* $(1+\epsilon)r$*. Then Algorithm Exp-Weight achieves ergodicity, for a suitable* $\alpha = \Theta(\epsilon)$*. Furthermore, the tail of the ergodic queue-size distribution at any link is exponentially decreasing with a decay rate* $\Omega(\epsilon)$*.*

Proof Since $q_i'(t) = q_i(t) + a_i(t)$ and $q_i(t)$ and $a_i(t)$ are independent,

$$\begin{aligned} \mathrm{E}\,[\phi(q_i'(t))] &= \mathrm{E}\,[\phi(q_i(t))]\mathrm{E}\,[\phi(a_i(t))] \\ &= e^{r_i(e^{\alpha}-1)}\mathrm{E}\,[\phi(q_i(t))]. \end{aligned}$$

Since there is an algorithm for which the system is ergodic under the arrival vector $(1+\epsilon)r$, $(1+\epsilon)r = \sum_k \alpha_k \chi_{S_k}$, where the α_k's are nonnegative and sum up to 1, as shown in Theorem 4.1. Given the vector $q'(t)$, assume that we schedule S_k with probability α_k at time t. If link i is nonempty, $q_i(t+1) = q_i'(t) - 1$ with probability $p_i = (1+\epsilon)r_i$. Thus,

$$\begin{aligned} &\mathrm{E}\,[\phi(q_i(t+1))] \\ &\quad \leq (1 - p_i + p_i e^{-\alpha})\phi(q'(t)) + 1 \\ &\quad = (1 - p_i + p_i e^{-\alpha})e^{r_i(e^{\alpha}-1)}\mathrm{E}\,[\phi(q_i(t))] + 1. \end{aligned}$$

A simple calculation shows that, for a suitable $\alpha = \Theta(\epsilon)$, the coefficient of $\mathrm{E}\,[\phi(q_i(t))]$ in the above equation is at most $1 - \Theta(r_i\epsilon^2) \leq 1 - \Theta(r_{\min}\epsilon^2)$, where $r_{\min} = \min_i r_i$. It follows that, if S_k is chosen with probability α_k,

$$\mathrm{E}\,[\Phi(q(t+1))] \leq (1 - \Theta(r_{\min}\epsilon^2))\mathrm{E}\,[\Phi(q(t))] + m. \tag{6.1}$$

The left-hand side of Equation 6.1 is a convex combination of the expected values of $\Phi(q(t+1))$ that we would have obtained by scheduling deterministically each of the S_k's. This implies that Algorithm Exp-Weight maintains the invariant in Eq. 6.1. We conclude that

$$\mathrm{E}\,[\Phi(q(t))] = O\left(\frac{m}{r_{\min}\epsilon^2}\right),$$

for all t. As a consequence,

$$\Pr\,[q_i(t) \geq B] \leq \frac{m}{r_{\min}\epsilon^2}e^{-\alpha B},$$

as desired. ■

We have experimented with the following heuristics. The first one was suggested in [19].

Heuristic Sequential: Initialize the subset S to be scheduled as the empty set. Order the links according to their weight and scan them in that order. If $S \cup \{i\}$ is feasible, let $S = S \cup \{i\}$.

Heuristic R-Sequential: Initialize the subset S to be scheduled as the empty set. For each remaining link i, let $R(i)$ be the set of remaining links j such that $S \cup \{i, j\}$ is not feasible. Add to S the remaining link i such that $w(i)/w(R(i))$ is maximum. Delete the remaining links j such that $S \cup \{j\}$ is not feasible.

Proposition 6.4. *When the feasible subsets are the independent sets of a graph* K*, the weight of the independent set obtained by Heuristic R-Sequential is at least the total weight divided by* $1 + \lambda_0(K)$*.*

Proof In the case where the feasible subsets are the independent sets of a graph K, the set $R(i)$ is the set of remaining links that are adjacent to i in K. Let s be the maximum ratio $w(i)/w(R(i))$ obtained in the first iteration. Thus, $w \leq sAw$, where A is the adjacency matrix of K. This implies that $s \geq 1/\lambda_0(K)$. The same bound holds in subsequent iterations, since the largest eigenvalue of a subgraph of K is upper bounded by $\lambda_0(K)$. Thus, the weight of the independent set I obtained by Heuristic R-Sequential is at least the weight of $K - I$ divided by $\lambda_0(K)$. This concludes the proof. ■

7. Implementation results

We present in this section the results of preliminary simulations of some of our algorithms. The mean delay is plotted versus offered traffic for each algorithm, under either the fixed discrete conflict graph model or with power control.

The graphs in Figures 1 through 4 are labeled using the following simple scheme:

(i) A leading s- indicates that the queue-sizes were used as the weights in the heuristics **Sequential** and **R-Sequential**, while a leading w- indicates that the waiting times were used.

(ii) A following pc- designation indicates that power control was employed, while a following ci- designation indicates that in the algorithm no power control was employed but the actual cumulative interference was determined using the path-gain matrix and assuming that all transmitter powers were identical.

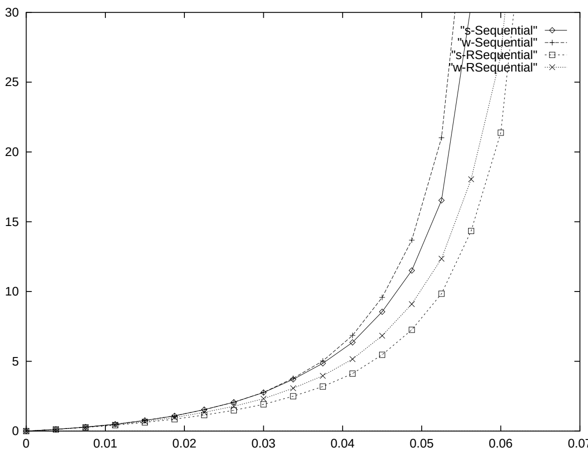

Figure 1: Mean delay versus offered traffic for 25 station, 150 link network at 1800MHz (PCS) in a large urban environment (discrete link conflict graph model).

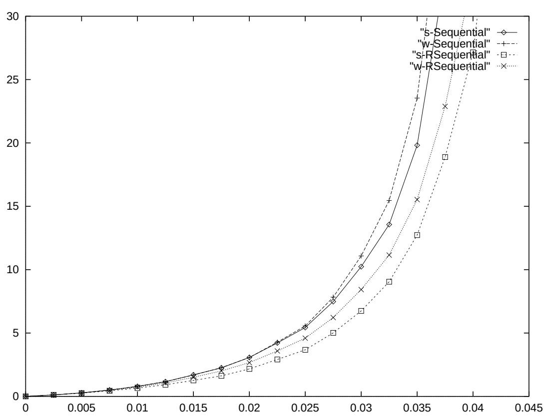

Figure 2: Mean delay versus offered traffic for 25 station, 150 link network at 900MHz (cellular) in a suburban environment (discrete link conflict graph model).

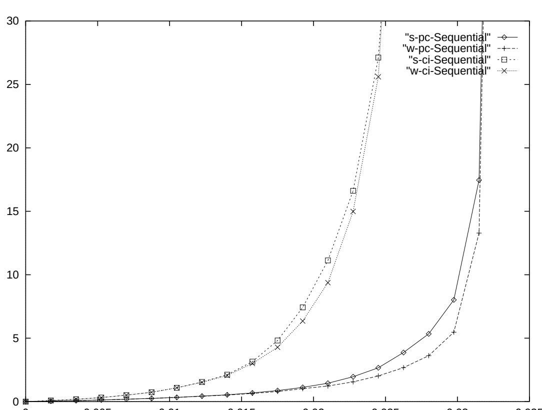

Figure 3: Mean delay versus offered traffic for 25 station, 150 link network at 900MHz (cellular) in a suburban environment with power control.

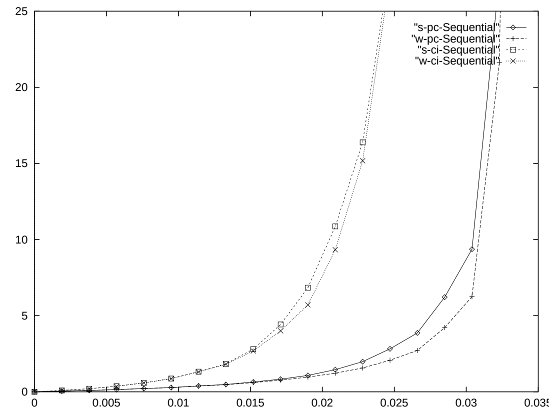

Figure 4: Mean delay versus offered traffic for 25 station, 150 link network at 1800MHz (PCS) in a suburban environment with power control.

Generally, our heuristics outperform those suggested in [19] in terms of mean delay. Disappointing results, in terms of mean delay, were obtained using the power control based algorithms versus the algorithms based on the discrete interference model. This can be explained by the fact that the latter algorithms, by simplifying the interference model, give worse signal-to-interference protection in general. The SIR losses of the conflict graph based algorithms need to be studied to quantify this effect.

7.1. Simulation model

We implemented the following simulation model:

1. 25 station locations were chosen at random points within a square area, 100 km per side.

2. Terrain models (urban for large, medium and small cities, suburban for open rural and quasi-open rural) were those of Hata [5, 3].

3. The average link length was in the 10km range. Links larger than 20 km were not considered due to the excessive path loss. This resulted in 150 links between pairs of stations.

4. Log-normal shadow fading with a standard deviation of 4dB was modeled as well but no multipath propagation effects were included in the study.

5. The conflict graph was constructed by assuming that a pair of links, i and j, could not operate simultaneously if $\min(G_{ij}, G_{ji}) <= 24$ dB.

6. In the case where actual path gains were used, a set of links was considered infeasible if the receiver on any link had an SIR less than 24 dB.

7. The arrivals at each link had independent identical Poisson distributions.

Because we consider here a single-channel system, in the conflict graph based model, a single station was explicitly ruled out from simultaneously transmitting to multiple sites.

7.2. Eigenvalue bounds

For the system corresponding to Figure 4, the eigenvalue bound technique of Section 5 yielded a bound of 0.244165 on the maximum stable packet arrival rate at each link. The over-estimate over the apparent maximum stable arrival rate under **Sequential** was by about a factor of 7. However it is not known how well the latter algorithm is performing compared to the optimum scheduling policy. The choice of the matrix B was not optimized for this calculation. The corresponding bounds for the systems illustrated in Figures 1 through 3 were 0.166932, 0.139353 and 0.244165 respectively.

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