Faster Pricing of Convertible Bonds

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discuss their latest pricing model for Convertibles

We study the pricing of convertible bonds using one-factor methods. Classical numerical algorithms based on lattice trees converge slowly as the number of time-steps increases when there is an issuer call. We describe an improved algorithm that has a much better rate of convergence. Our algorithm also evaluates accurately other options having a trigger level, such as barrier options. The algorithm is robust and yields smooth prices and Greek parameters as the spot approaches the trigger level. It is simple to implement and has essentially the same running time as a lattice tree algorithm.

The classical algorithm

We first describe the classical way to price convertible bonds with lattice trees. For simplicity we use binomial trees throughout the paper, but the extension to trinomial trees is possible. We also assume that the tree has been constructed so that up and down movements occur both with probability 0.5.

Consider a convertible bond with an issuer call that pays coupons at predetermined dates. At a given node *i, j,* let $\overline{C}$ be the continuation value computed from the discounted average value of its two children, taking into account coupons if there are any between time-steps i and $i+1$. The value at node *i, j* if the bearer has no right to convert at time-step i is equal to $\overline{C}$. If the bearer has the right to convert at time-step i and the issuer call is not valid at the node, the value at node *i, j* is max ($CV_{i,j}$, $\overline{C}$), where $CV_{i,j}$ is the conversion price. If the issuer call is valid at the node, the value at node *i, j* is max ($CV_{i,j}$, min (R_i, $\overline{C}$)), where R_i is the call price.

The issuer call is in general valid in certain time periods if the spot is above a trigger level. In such a case, the above algorithm has a low rate of convergence, as illustrated in Fig. 3. This is because the trigger level of the issuer call does not coincide with tree levels, and the spacing between tree levels decreases slowly compared to the time-step. This point is explained in detail in the case of barrier options in several papers such as [BL94,DKEB95].

The improved algorithm

The improved algorithm is in the same spirit as the one described in [DKEB95] to price barrier options. Our algorithm uses just one pass, however. For simplicity, we first describe it in the case of an up-and-out barrier option. In the improved algorithm, we compute the value of the option at time-steps i for decreasing values of i in the usual way. At each time-step, however, we check whether there exists a node *i, j* such that the spot $s = s_{i,j}$ is below the barrier level H and $s_1 = s\sqrt{u/d}$ is above the barrier, where u and d are the up and down movements multiplication factors. If such a node *i, j* exists we multiply the option value at node *i, j* by $\lambda = (H - s)/(s_1 - s)$ if i is the last time-step, and by $2\lambda/(1+\lambda)$ otherwise. We observe that such a node *i, j* does not exist for all time-steps but for roughly half of them. This algorithm can be easily adapted to double barrier options. In Fig. 1, we compare the prices given by the Cox and the Improved Cox algorithms to a semi-analytic price for a double barrier option given in [S98]. In both algorithms we compute the average price corresponding to two successive numbers of iterations.

Fig. 1

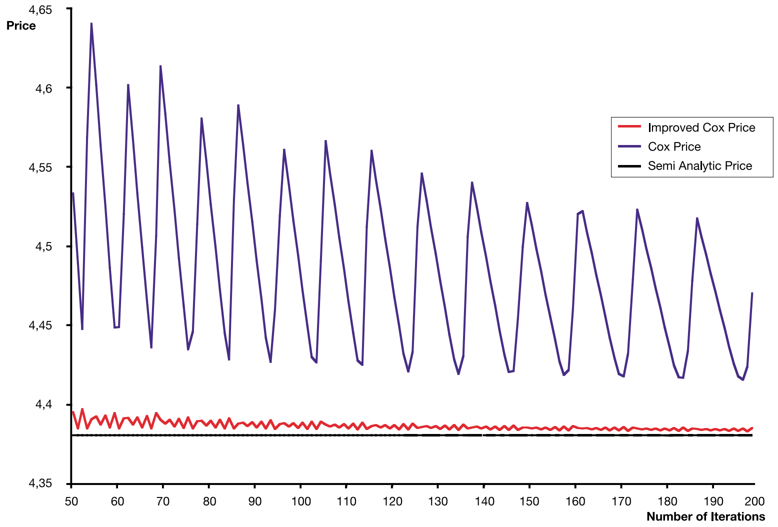

Fig. 1. Cox and Improved Cox prices in terms of the number of iterations for a double barrier call with initial spot S_0 = 100, strike K = 100, volatility σ = 25%, interest rate r = 10%, dividend rate q = 5%, upper barrier H = 130, lower barrier L = 75, and time to maturity T = 0.25. The semi-analytic price given in [S98] is 4.3806. The Improved Cox price for 3200 iterations is 4.3807.

Fig. 2

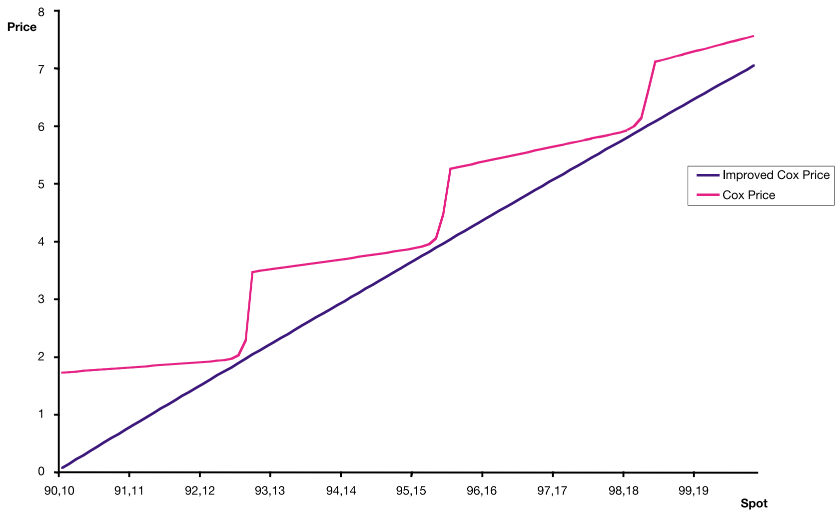

Fig. 2. Cox and Improved Cox prices as the spot approaches the barrier for a down-and-out call with strike K = 110, volatility σ = 30%, interest rate r = 5%, no dividend rate, barrier L = 90, and time to maturity T = 1. The number of iterations is fixed at 100.

In Fig. 2, we compare the prices given by the Cox and the Improved Cox algorithms for a down-and-out call as the spot approaches the barrier.

Theoretical justification

The choice of the factor $2\lambda/(1+\lambda)$ is motivated by the following. We assume in this argument that the barrier holds throughout the life of the option and there are no discrete dividends, but experimental results show this assumption is unnecessary. Near the barrier, the theoretical value of the option $u(s,idt)$ at node i, j is approximated by the discounted value of $u(s,(i+1)dt)$. Moreover, by linear interpolation,

$$u(s,(i+1)dt) \approx \frac{H-s}{H-s_{i+1,j+1}} u(s_{i+1,j+1},(i+1)dt).$$

It follows that the value of u at node i, j is approximated by the average discounted value of its two children multiplied by $2(H - s) / (H - s_{i+1,\,j+1}) \approx 2\lambda/(1+\lambda)$.

Note that the multiplication factor tends to 1 as s_1 tends to the barrier from above and to 0 as s tends to the barrier from below, ensuring that the price is continuous in terms of the spot. If we used the multiplication factor λ instead of $2\lambda/(1+\lambda)$ the resulting algorithm would still be continuous, but experimental results show it has a lower rate of convergence than our algorithm.

Pricing of Convertibles

Again, our algorithm for convertibles pricing is a modification of the classical algorithm, and proceeds by computing the value of the option at time-steps i for decreasing values of i in the usual way. Using the same notation as before, we check at each time-step whether there exists a node i, j such that the spot $s = s_{i,j}$ is below the trigger level H and s_1 is above the trigger level. If such a node i, j exists we let C_1 be the value of the option at node i, j computed by backward induction in the usual way, and C_2 the value of the option if the issuer call were active at node i, j. The final value we assign to node i, j is $C = \alpha C_1 + (1-\alpha)C_2$, where $\alpha = \lambda$ if i is the last time-step and $\alpha = 2\lambda/(1+\lambda)$ otherwise.

In Fig. 3, we compare the prices given by the Cox and the Improved Cox algorithms for a convertible bond in terms of the number of iterations.

Fig. 3

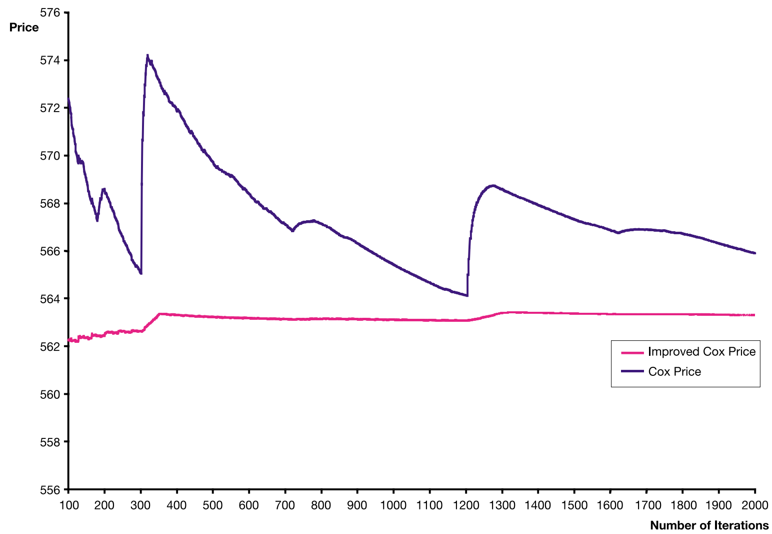

Fig. 3. Cox and Improved Cox prices in terms of the number of iterations for a convertible bond with initial spot $S_0 = 550$, nominal value $N = 500$, volatility $\sigma = 45\%$, interest rate $r = 10\%$, yearly discrete dividend $d = 10$, yearly coupon $C = 6\%$, issuer call trigger level $H = 580$, issuer call redemption value $R = 500$, issuer call valid on the entire convertible bond lifetime and time to maturity $T = 6$. The number of iterations ranges from 100 to 2000.

In Fig. 4, we compare the prices given by the Cox and the Improved Cox algorithms for a convertible bond as the spot approaches the trigger level

Fig. 4

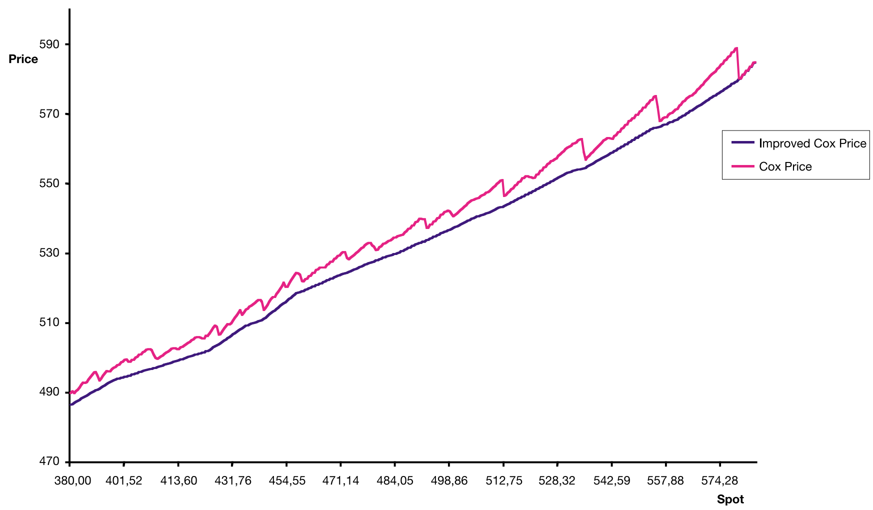

Fig. 4. Cox and Improved Cox prices as the spot approaches the issuer call trigger level for the same convertible bond as in Fig. 3. The number of iterations is fixed at 500. The spot price ranges from 380 to 580.

Fig. 5

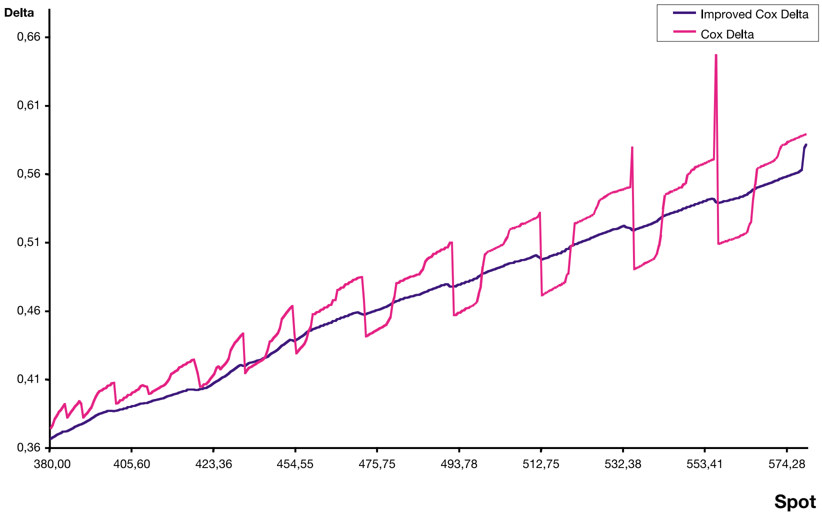

Fig. 5. Cox and Improved Cox deltas as the spot approaches the issuer call trigger value for the same convertible bond as in Fig. 3. The number of iterations is fixed at 500. The spot price ranges from 380 to 580.

In Fig. 5, we compare the deltas given by the Cox and the Improved Cox algorithms for a convertible bond as the spot approaches the barrier.

Summary

Our algorithm for pricing convertible bonds with an issuer call converges much faster than the usual tree algorithms with almost no additional cost in running time. For a given number of iterations it yields a continuous price as a function of the spot. It also gives accurate and meaningful hedging parameters even as the spot approaches the trigger level. This algorithm is implemented in the Sophis RISQUE software.

References

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